The Logic of Content as and its Applications

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Abstract

The purpose of this project is the development of a *logic of content*, namely a logic based on the fundamental idea that the relation of entailment is a relation of content containment. The logic of content is expected to improve the existing systems of relevant logic on three aspects: 1) it is based on a fundamental semantic idea rather than on ad hoc limitations on the deductive system, 2) It is more in line with scientific and mathematical reasoning, 3) in addition to the paradoxes of material implication, it also deals with the *fallacies of relevance* associated with Hempel's paradoxes of confirmation. As a working hypothesis, it is assumed that the notion of content expressed by a sentence S is framed in terms of the way S is verified or falsified. The project is also concerned with possible applications of the logic of content to metaphysics, in particular the development of a logic of truth-making and grounding.

Resumo

O propósito deste projeto é o desenvolvimento de uma *lógica do conteúdo*, isto é, uma lógica baseada na ideia fundamental da relação de consequência como inclusão de conteúdo. A espectativa é que a lógica do conteúdo represente um melhoramento das lógicas relevantes pelo tocante a três aspectos: 1) é baseada em uma ideia semântica fundamental e não em restrições ad hoc do sistema dedutivo; 2) é mais compatível com o raciocínio matemático e científico; 3) além de lidar com os paradoxos da implicação material, a lógica do conteúdo deveria evitar também as falácias de relevância devidas aos paradoxos da teoria da confirmação de Hempel. A hipotése de trabalho é definir A noção de conteúdo expressado por uma sentença S em termos da maneira em que S é verificada ou falsificada. O projeto prevê também possíveis aplicações da lógica do conteúdo para a metafísica, em particular o uso da lógica do conteúdo para o desenvolvimento da lógica das relações de *truth-making* e *grounding*.

1 Introduction

Classical logic has been widely criticized for allowing "fallacies of relevance" as valid formulas. Such criticism started with MacColl (1908) who considered the validity of the following formulas as counter-intuitive: for any two propositional variables p, q,

(A) $\vDash p \to (q \to p)$

(B) $\vDash (p \wedge \neg p) \to q$

(C) $\vDash p \to (q \vee \neg q)$

Formulas (A)-(C) are commonly considered as paradoxes of material implication. Clearly, the word paradox is used in its original and etymological meaning, i.e. as something that is in conflict with certain intuitive and widely acceptable principles. Recalling that the meta-theorem of deduction holds in classical logic, (A)-(C) may be interpreted also as "paradoxes of classical consequence", i.e.

(A′) $p \vDash (q \to p)$

(B′) $p, \neg p \vDash q$

(C′) $p \vDash (q \vee \neg q)$

Why should we consider (A′)-(C′) as fallacies of relevance? One may assume that propositional variables are mutually independent and that notions such as consequence and implication should capture the fact that the premises have a certain "grip" on the truth-value of the conclusion. Yet how the truth of p is supposed to have a certain grip on the truth of e.g. $q \rightarrow p$ given that p says nothing about q? Why should we accept an argument assuming that Socrates is wise and concluding that Plato is Greek implies that Socrates is wise as valid? After all no one would ever make such an inference in an argumentative context; moreover – and more significantly – such an inference schema could never be used in scientific or mathematical reasoning.

The problem of accepting fallacies of relevance as valid inferences is not limited to the relation between formal and applied reasoning. There are difficulties that arise with the emerging conception of logic itself. Consider case (B′): such an inference is commonly known as *the principle of explosion*, or *ex falso quodlibet* as the medieval scholars labeled it. By accepting (B′) as a valid inference, we allow that every formal system that proves a contradiction is thereby trivialized, i.e. it admits every well formed sentence as valid. And it is by no means clear why contradictions should have such a devastating effect on formal systems.

These considerations have been traditionally considered as the starting point of the research in *relevant logics* which can be dated back to the work of Anderson and Belnap (1975). Roughly speaking, an example of the relevantist approach may be described as follows: in order to avoid the paradoxes of material implication (and thus those of classical consequence) some deduction rules should be abandoned. For instance, if we consider a proof of (B) in natural deduction, we see that three rules are used: the introduction of the disjunction, the *modus tollendo ponens*, and the introduction of material implication. Revalantists reject the *modus tollendo ponens*, i.e. the rule that from two assumptions of the form $p \vee q$ and $\neg q$ allows to infer q.

A great deal of work has been done in the development of relevant logics in the last fifty years. Anderson and Belnap (1975) developed axiomatic systems **R** and **E** for relevant logic; formal semantics are developed in (Urquhart, 1972), (Routley et al., 1982), and (Fine, 1974); a philosophical interpretation of relevant logic is provided in (Mares, 2007) and (Dunn and

Restall, 2002); first-order extensions of the main systems for relevant logic has been developed (for a survey of the most recent results see (Jago, 2013)).

To what extent relevant logic has solved the so-called paradoxes of material implication and classical consequence?

From the perspective of the calculus, formulas like (A),(B), and (C) are not in general valid in the main systems of relevant logic. However, given that the need for a relevant logic stems from a philosophical concern on the fundamental notions of logic like implication and entailment, the definition of a new calculus may not be considered as a satisfactory solution to the aforementioned paradoxes. This because we do not need just a calculus that avoid the paradoxes, but rather a system of deduction and a correspondent semantic theory that vindicates our basic intuitions on how entailment and implication should behave.

When we consider relevant logic from such a wider perspective, we cannot conclude that a satisfactory answer to the paradoxes of material implication and classical consequence has been provided. In particular, we may point out two fundamental difficulties in considering relevant logic as a solution to the paradoxes: 1) the fact that the solution proposed by the relevantists is composed of a series of ad hoc limitations to classical deduction; 2) the fact that relevant deductions do not always seem to be the formal counter-part of scientific and mathematical deductions.

Regarding 1), it is clear that the relevantist approach may be reduced to the rejection of all modes of inference that lead to fallacies of relevance. Yet no independent reasons are given to reject those modes of inference. For instance, why should we reject modus tollendo ponens? After all – as remarked by Tennant (2005) – this mode of inference turned out to be a fundamental tool in mathematical and scientific reasoning. This latter fact leads to point 2): given that classical logic is "relevantized" by using ad hoc limitations to the deductive system and without providing independent and general reasons that are in accordance with our fundamental intuitions regarding logical notions, we cannot expect the resulting relevant deductive system to be a satisfactory representative of our natural way of reasoning in mathematics and science which surely proceeds according to such intuitions.

The issue may be presented from a different standpoint. From the fact that the main systems of relevant logic are obtained by introducing ad hoc assumptions on the valid modes of infer-

ence, it follows that there is no clear semantic idea behind the relevant notion of entailment. For instance, we define classical consequence as preservation of truth: if a sentence ψ is a classical consequence of a sentence ϕ, then it is impossible that ϕ is true and ψ false. Preservation of truth is the fundamental semantic idea behind the notion of classical consequence. What is the correspondent idea for relevant consequence? There does not seem to be a unitary and defined answer to this question.

One may suppose that relevant consequence preserves both truth and a certain unspecified notion of subject matter. However, under any reasonable account of what the subject matter of a sentence is, this is not case for the main systems of relevant logic. This is due to the fact that relevant logic admits the introduction of the disjunction as a valid inference. For instance, 'Socrates is wise or it is raining in Lichtenstein' is a relevant consequence of 'Socrates is wise'; yet the subject matter of the consequent contains a part that is not included in that of the antecedent, i.e. the weather in Lichtenstein.

A difficulty in understanding to what extent relevant logic is apt to model scientific reasoning arises from its application to the so called paradoxes of confirmation ((Hempel, 1946), (Hempel, 1983), (Hempel, 1945)). The paradoxes of confirmation are interesting when relevant logic is concerned in virtue of the fact that they may be conceived as engendered by certain fallacies of relevance. According to Hempel's account, a sentence S confirms a general scientific law expressed by a sentence L if S entails an instance of L. The classical example is the sentence 'Edgar is black and Edgar is a raven' that confirms the law 'All ravens are black' in virtue of the fact that 'Edgar is black and Edgar is a raven' entails 'If Edgar is a raven then Edgar is black'. The paradox of confirmation arises when we combine Hempel's account with Nicod's equivalence condition which says that given a scientific law expressed by a sentence L and given L' logically equivalent to L, if S' entails an instance of L', then S' confirms L. Now given that 'All non-black things are not ravens' is equivalent to 'All ravens are black' and that 'Bart is a white rabbit' entails 'If Bart is not black, then Bart is not a raven', 'Bart is a white rabbit' confirms 'All ravens are black'. This conclusion may taken as paradoxical if we consider that 'All ravens are black' seems to be about black ravens and not about white rabbits. It is not immediately clear in which sense a white rabbit should confirm that all ravens are black, since white rabbits do not seem to be relevant to a law asserting the blackness of all ravens.

A natural reply to the paradox may be that classical entailment is not adequate to characterize the relation of confirmation, for it is not sensitive to changes in subject matter and thus introduces certain fallacies of relevance. Is relevant logic a good candidate to replace classical logic in characterizing the relation of confirmation and avoid the correspondent paradoxes? The answer is negative. This because relevant implication obeys to the law of contraposition precisely as material implication does. Therefore, the paradox persists when we formalize 'All ravens are black' using a relevant implication instead of material implication.

It is worth taking a stock. In spite of both their development as calculi and their applications, the main systems of relevant logic may still be considered as unsatisfactory under the following respects:

- The paradoxes of material implication and classical consequence are avoided by introducing ad hoc assumptions regarding the valid modes of inference,
- There is no clear semantic idea behind the notion of relevant entailment,
- Some modes of inference (e.g. modus tollens) rejected by the relevantists seem to be essential to mathematical and scientific reasoning,
- Both relevant entailment and implication are not suitable to characterize an improved version of the Hempelian relation of confirmation.

The purpose of this project is the definition of a new approach to relevant logic that improves on the traditional one by dealing with the four aforementioned difficulties. The guiding idea is to introduce (or perhaps re-introduce) in logic a certain notion of content and to define relevant entailment as a relation of content containment.

2 Exposition of the problem

As briefly mentioned in the previous section, the fundamental idea behind this project is the definition of a new approach to the problem of the fallacies of relevance based on the introduction of the notion of content into logic and on the definition of entailment as content containment. We expect to define new logical systems that are based on this fundamental idea and characterized by a notion of entailment applying requirements on relevance stricter than those applied

by the entailment of relevant logic. In order to distinguish the proposed approach from the relevantist approach of Anderson and Belnap's tradition, I will speak of relevant logic when the latter is concerned and *logic of content* when the former is concerned.

As suggested in the previous section, there are four main respects under which the logic of content is expected to improve on the relevantist approach: the logic of content should be based on a central semantic idea regarding the notion of entailment and not on ad hoc assumptions regarding the deductive system; modes of inference commonly used in mathemtaical and scientific reasoning should be allowed; the logic of content should be a suitable candidate to replace classical logic in a Hempelian theory of confirmation.

The proposal encounters an immediate difficulty: **How the content expressed by a sentence may be defined in a formal semantic setting?**

It is worth noting that a certain notion of content was considered as a fundamental semantic unit in the earliest stages of the development of logical calculus. Frege himself in the beginning sections of his *Begriffsschrift* introduces the notion of *conceptual content* (Begriffinhalt) as what is responsible for the determination of all consequences of a given sentence (Frege, 1967). However, Frege quickly abandoned this approach, probably due to the difficulty in treating content in an extensional way. An attempt to define an extensional notion of content may be to identify the content expressed by a sentence with its *truth-value profile*, i.e. as the set of interpretations, or models, or states, or possible worlds in which the sentence is true. This leads to a notion of content close to Carnap's notion of *intension* (Carnap, 1947). Yet we immediately see that if interpretations, models, states, and possible worlds are defined according to standard formal semnatics (e.g. Tarskian semantics, Kripke's semantics, Carnap's semantics) the resulting notion of content does not grant the sought grade of relevance: for all necessary truths have the same truth-value profile, yet to say that they share the same content seems to allow a great deal of fallacies of relevance: if '2+2=4' expresses the same content as 'Socrates = Socrates' then 'Socrates = Socrates' must be a relevant consequence of '2+2=4' (and vice versa). Salmon (1992) attempts a refinement of this approach by introducing what he calls *propositional models*; yet it is not clear how to implement a formal semantics based on this notion.

In this project we choose a different approach. Given that fallacies of relevance may be

understood as arbitrary change in subject matter, whatever the content expressed by a sentence is, it must encode enough information regarding the subject matter of the sentence. The basic idea is suggested by Yablo's proposal in defining the notion of exact subject matter of a sentence in terms of ways of being true (false) (Yablo, 2014). We make the following working hypothesis:

Working Hypothesis: *Given that the intension of a sentence tells when (i.e. in which cases) a sentence is true or false, in addition to that the content of a sentence tells how it is true or false* .

What should be counted as a way of being true or false of a sentence? According to Yablo (2014) a way of being true (false) is some wide conception of truthmaker (falsemaker). Thus we may further develop our working hypothesis (1) and assume that the content of a sentence is represented by both its truth-value τ in a certain interpretation, model, state, possible world, and the entities responsible for the sentence having τ as truth-value. If we allow – for simplicity – the jargon of possible worlds, we may say that given a possible world w, the content of a sentence S should tell us both the truth-value of S in w and the set of verifiers or falsifiers of S in w.

Thus the notion of content that is to be defined is a sort of objective – or "real" as Yablo calls it – non-representational content.

The immediate consequence of our **Working Hypothesis** is that the notions of verification and falsification are crucial for the logic of content and represent the main theoretical enrichment of this logic compared to classical logic (which is interested just in the truth-value of a sentence). This leads to the following question: **How the notions of verification and falsification should be implemented in a system of logic of content?**

In many cases in which a new system of logic is aimed at characterizing new theoretical notions, the chosen approach depends on whether such new notions are to be characterized syntactically or semantically. Thus we have two different approaches:

- *Syntactical approach:* Given a sentence S the content of S and the relations that this content bears to contents of different sentences may be treated syntactically, i.e. by introducing new syntactical resources in the language of propositional logic. For instance, we may enrich a propositional language with a suitable category of expressions standing for verifiers or falsifiers as well as by introducing primitive relational symbols that apply to

this new class of expressions and sentences standing for the relations of verification and falsification,

- *Semantic approach:* We may define an "unorthodox" semantic for a propositional language countenancing in addition to valuation functions domains of individuals that are possible verifiers or falsifiers of the sentences of the language. Relations of verification and falsifications are recursively defined by suitable semantic clauses starting from primitive relations of verification and falsification. In this case a model for the considered language is composed of at least four ingredients: a valuation function, a domain, and two primitive relations between elements of the domain and propositional variables (i.e. verification and falsification).

The syntactic approach presents the difficulty of increasing the complexity of the calculus: given the extension of a propositional language required by this approach, new axioms and inference rules are required to govern the behaviour of the new expressions according to the fundamental idea that the content of a sentence is related to the way it is verified/falsified. On the other hand, the semantic approach introduces minimal changes in the calculus but at the cost of using a very complex semantics. Moreover, once the notions of semantic consequence and validity are defined in such a new semantics, the resulting logical system may present the conceptual problem of being interpretable in many different ways, and it may fail to capture the fundamental idea behind the project.

Once the general lines along which the notion of content is to be defined are traced, the next step is to define the correspondent notion of entailment. As anticipated in the previous section, the fundamental semantic idea behind the logic of content is the notion of entailment as content containment. This leads to the following question: **How the notion of entailment as content containment may be defined?**

The idea of defining a logic of entailment as containment is not a novelty. In a series of works ((Angell, 1989), (Angell, 1977)) Angell proposes the system **AC** intended to be a logic of analytic containment. It is interesting to note that recently Angell's system has gained a great deal of attention insofar as it may be interpreted as a logic of truth-making or factual equivalence ((Fine, 2016), (Fine and Jago, 2018), (Correia, 2004), (Correia, 2016)). As in the case of systems **R** and **E**, Angell's logic **AC** does not stem from a general semantic definition

of entailment as containment; Angell defines his system rather by developing some syntactic considerations. However, Angell's proposal does have a central idea which may be called *the conjunctive pattern of content containment*; roughly speaking, the conjunction is intended as a sort of content merging operation. Thus if the content of a sentence B is included in the content of A, then A has the same content as a conjunction having B as one of the conjuncts. Such an idea is implemented by admitting identity of content $\equiv$ as a logical connective[1]; Angell's implication $A \rightarrow_{\mathbf{AC}} B$ is defined as

$$A \equiv (A \wedge B)$$

In spite of the fact that the conjunctive pattern of content containment is attractive from a theoretical point of view, Angell's system **AC** cannot be considered as a main reference for the development of the logic of content. This due to the fact that the deductive system of **AC** is arguably far from being a formal model of scientific reasoning: indeed, all theorems of **AC** have the form of an identity of content and the only inference rule is – roughly speaking – the preservation of content under substitutions of equivalent expressions.

However, as it will be shown in the next section, the general idea of the conjunctive pattern of content containment may be retained in the context of a verificational conception of content.

The main difficulty related to the definition of a relation of consequence as content containment between two sentences S and T is how such a relation of containment may be framed in terms of verifiers/falsifiers respectively of S and T .

The conjunctive pattern offers a preliminary answer: all falsifiers of A are also falsifiers of $A \wedge B$; yet the same does not occur if we consider the verifiers. Indeed, it seems that all verifiers of A are *partial verifiers* of $A \wedge B$, in the sense that all verifiers of $A \wedge B$ are the result of some sort of combination of the verifiers of A and those of B. Therefore, if the conjunctive pattern is to be implemented, there are two possible way to proceed to formulate a general definition of content containment: either we presuppose primitive operations of combinations (e.g. mereological, set theoretical, etc...) between verifiers/falsifiers, or we presuppose primitive relations of partial verification/falsifications. Fine (2016) represents an example of the first option: his truth-maker semantics for Angell's logic **AC** is based on primitive formally mereological relations between "states", where states are taken to be primitive verifiers/falsifiers of atomic sentences. However,

[1]The propositional signature of **AC** is composed of negation, conjunction, and identity of content.

given that we should not limit our analysis to the conjunctive pattern of content containment, the second option – i.e. the introduction of relations of partial verification and falsification – may provide more flexibility in developing alternative accounts of content containment. Such alternatives are worth exploring given that the conjunctive pattern seems to impose too strong requirements on the relation of consequence (for this reason Angell's system is very weak) and less severe consequence relations may strengthen the resulting deductive system.

A last difficulty is related to the notion of *implication* that a logic of content is expected to introduce. Surely such an implication should avoid the paradoxes of material implication; in this sense, the logic of content has the same purpose of relevant logics. However, given that in the case of the logic of content we do have a fundamental semantic idea regarding the notion of entailment, the following difficulty arises: **What is the relation between the notion of entailment as content containment and the notion of implication of the logic of content?**

In the case of classical logic, classical consequence is defined as preservation of truth in all models, while material implication as preservation of truth in one model. In other words, material implication may be seen as the "local counterpart" of classical entailment. **Should the implication of the logic of content be the "local counterpart" of content containment? How are we expected to understand relations between contents limited to a single interpretation, model, possible world?**

Another problem related to the definition of an implication within the logic of content arises when we consider the paradoxes of confirmation. It has been mentioned that such paradoxes may be considered as additional fallacies of relevance and a logic of content may represent an improvement on relevant logic precisely in the sense that it blocks the paradoxes of confirmation. From a logical point of view, it is evident that the paradoxes of confirmation arises due to the law of contraposition of material implication. As a consequence, **the development of a logic of content faces the difficulty of defining a new notion of implication that does not obey to the law of contraposition in all cases**.

A notion of implication compatible with the provided general requirements on the logic of content engenders further difficulties also on the semantic level. For, as we have seen, a semantics for the logic content should not deal just with the truth-values of the sentences of the language but also with their verifiers/falsifiers. Thus several problems arise with the ascription

of verifiers/falsifiers to a sentence of the form $A \Rightarrow B$ where $\Rightarrow$ is the implication of the logic of content. For instance, **How an implication that is false due to a lack of relevance is falsified? Which are its falsifiers?** Another problem – strictly related to the avoiding of the paradoxes of confirmation – is the behaviour of an implication having a false antecedent. Indeed, in this case we consider the implication as *vacuously true*; however, the assumption that the falsifiers of the antecedent play a role in the verification of the implication gives rise to the paradox of confirmation. Thus the following question arises: **Does a vacuously true implication have verifiers?** The idea that an implication having a false antecedent is "true for no reason" is suggested by Yablo (2014). It is worth exploring this theoretical possibility in the development of the logic of content.

3 Methods and Aims

3.1 Methodology

As highlighted in the previous section, the problem of the development of a logic of content faces the following difficulties:

(D_1) The problem of defining the content of a sentence S in terms of the verifiers/falsifiers of S in each interpretation, model, possible world,

(D_2) The problem of defining an entailment as content containment between two sentences S and T in terms of a relation between the verifiers/falsifiers of S and those of T,

(D_3) The problem of defining an implication in accordance with the notion of entailment.

As previously mentioned, there are two main approaches to deal with (D_1): the syntactic approach and the semantic approach. This project foresees the evaluation of both. Regarding the syntactic approach, an interesting possibility is the application of a particular interpretation of *justification logic* (Artemov, 2008). Justification logic is a family of systems whose language is a propositional language endowed with *justification terms* and *operators on justifications*. In the language of justification logic the expression $t : S$ is taken to mean 't is a justification for believing/knowing that S'; justifications may be combined in two ways: by "nesting" them

through the product operator '$\cdot$', or by "aggregating" them through the sum operator '$+$'. We propose a particular interpretation of justification logic to the effect that '$t : S$' is read as 't is a verifier of S'. It has to be evaluated whether falsifiers of S may be introduced as the application of justification terms to $\neg S$, or if an additional application operator is needed. Operations on justifications will be used to recursively define verifiers/falsifiers for molecular sentences in terms of the verifiers/falsifiers of their propositional variables. Moreover, to syntactically define a relation of content inclusion we will probably need to quantify over "justifications"; to this end, part of the project will be dedicated to the development of the logic of content in the framework of quantified justification logic (Fitting, 2008). The use of the resources of justification logic is attractive also because it locates the logic of content in the general context of the logics of provability and of epistemic logics.

Regarding the semantic approach, an unorthodox notion of model may be defined along the lines roughly described in the previous section.

Regarding the difficulty (D_2), we will explore at least two different definitional patterns of content containment: *the conjunctive pattern*, and what we may call *the partial witnessing pattern*. The partial witnessing pattern may be defined using primitive relations of partial and total verification/falsification and is based on the working hypothesis that the content of B is included in the content of A iff in all models whatever totally verifies (falsifies) B, partially verifies (falsifies) A. The partial witnessing pattern seems promising insofar as defines a relation of consequence stronger than that defined according to the conjunctive pattern. For instance, it is easy to check that the modus tollens is not always a valid inference according to the conjunctive pattern whereas seems to be valid according to the partial witnessing pattern. According to the way we define models including verifiers/falsifiers for the propositional language at issue, more patterns of content containment may be introduced. As a consequence, several systems of logic of content may be defined.

Once the entailment as content containment is defined, several axiomatizations of the logic of content will be defined taking into account which inferences are valid or not according to the defined entailment. The next step will be the proofs of soundness and completeness of these systems.

Regarding (D_3), different implications may be defined according to the way the relation

of content containment is defined. As upshot of the syntactic approach, we expect content preserving implication to be defined as a derivative connective in justification logic in terms of material implication with suitable restrictions formulated in terms of partial preservation of verification/falsification. More implications may be defined within the semantic approach. To deal with the problem of the paradoxes of confirmation, one possibility is to define an implication in a four valued semantics, by allowing the following truth-values: true and verified, vacuously true, vacuously false, false and falsified. The use of a four valued semantics will offer the occasion to evaluate possible theoretical connections between the logic of content and Nelson's logic of constructive negation (Nelson, 1949), which is one of the most interesting case of a logic presented with a four-valued semantics. Such connections may be grounded in a constructive interpretation of the logic of content: indeed, a sentence T is deducible from a sentence S in the logic of content only if all ways of being true (false) of S are determinable given the ways of being true (false) of T. Therefore the logic of content may be interpreted as a logic of constructive verification and falsification.

3.2 Aims

The project is aimed at developing several systems of logic – preliminary labeled *logic of content* – based on different definitions of consequence as content containment; in particular, the purpose of these systems is to improve the already existing systems of relevant logic **R** and **E** under the following respects:

- The development of a deductive system based on profound semantic intuitions regarding the notion of consequence,
- Such a system may allow for inferences that are commonly used in mathematical and scientific reasoning,
- Implication and consequences as defined within the logic of content should not engender the paradoxes of confirmation to the extent that these paradoxes are conceived as fallacies of relevance.

The logic of content should prove its fruitfullness through its applications, especially in the fields of philosophy of science and metaphysics. In particular, an interesting test for the systems

of logic of content is their use as underlying logic for Hempel's confirmation theory. In light of this sort of application, the possibility of defining first-order extensions of the logic of content will be evaluated. Moreover, I envision a possible application of the logic of content also to metaphysics: indeed, given the expected connection between the logic of content and Angell's logic **AC**, and given Fine's intepretation of **AC** as a logic of truth-making, some versions of the logic of content may be used to develop a logic of truth-making and, possibly, a logic of the relation of grounding.

4 Phases

The project is divided in four phases. The duration of each phase is approximately of six months.

4.1 Phase 1: definition of the conceptual machinery

In this phase the fundamental definitions of content, content containment, and validity will be formulated. The aforementioned variety of approaches and definitional patterns are expected to give rise to different definitions.

4.2 Phase 2: development of the formal systems

In this phase several axiomatizations of the logic of content will be developed. We expect to have systems defined in the language of the logic of justification and systems defined in the classical propositional language. Proofs of soundness and completeness are envisaged for all systems. The systems will be evaluated also according to different semantics, e.g. Routley's semantics for relevant logic, Kripke frames, intuitionistic Kripke frames.

4.3 Phase 3: connections with different logics

In this phase the already developed systems of the logic of content will be compared with the logics of provability and with Nelson's logics. Moreover, the possibility of expressing the logic

of content in the language of paraconsistent logics developed in (Carnielli and Coniglio, 2016) will be evaluated.

4.4 Phase 4: applications

In this phase the logic of content will be tested by considering main applications to the philosophy of science and metaphysics. First-order extensions of the systems developed in phase 2 will be considered. The Hempelian notion of confirmation will be implemented by using the notions of entailment and implication of the logic of content. Regarding the application to metaphysics the logic of content will be used as underlying logic of possible systems of the logic of truth-making and grounding.

5 Timetable

Phase 1: (*First Semester*) Analysis on the relevant literature on the notion of propositional content from a logical perspective. Formulation of a definition of content containment according to both the conjunctive and the partial witnessing pattern in different semantic settings. Possible alternative definitional patterns will be considered. Different definition of implication will also be formulated. During this phase I expect a frequent interaction with Prof. Edélcio de Souza given his work on the notion of consequence and implication (especially regarding the algebraic properties of the relation of consequence) and his experience in model theory. I also expect to interact with Prof. Rodrigo Bacellar given his experience in the foundations of logical notions and in the relation between logic and metaphysics.

Phase 2: (*Second Semester*) In this phase I expect to develop several axiomatic systems for the logic of content. As previously specified, some systems will be developed in the language of justification logic, while some others in the classical propositional language. The next step will be the proof of soundness and completeness for the defined systems.

Phase 3: (*Third Semester*) In this phase I will analyze possible connections between the logic of content and other logics, in particular other paraconsistent non-dialetheic logics as Nelson's logic **N4**. In this phase I expect a more frequent interaction with Prof. Edélcio G.

de Souza given his skills in non-classical logics. Moreover, I expect also to have a deep interaction with Prof. Giorgio Venturi and Prof. Marcelo Coniglio from UNICAMP given the deep interaction I had with them during my PhD at the Department of Philsophy at UNICAMP.

Phase 4: (*Fourth Semester*) In this phase the logic of content will be applied both to Hempel's confirmation theory and to the logic of truth-making and grounding. I expect to benefit from the interaction with Prof. Osvaldo Pessoa – given his interest in the philosophy of science – and Prof. Rodrigo Bacellar given his research in the relations between grounding and entailment.

Regarding this phase, I also consider an optional extension envisioning the possibility of spending one or two semesters in a foreign institution In particular, the application of the logic of content to metaphysics may be deepened by working with Prof. Kit Fine at New York University and Prof. Mark Jago at Nottingham University.

6 Expected outcomes

I expect to benefit from the interaction with the mentioned faculty members at the department of philosophy of São Paulo University (USP).

In particular, I expect to achieve the following outcomes:

- Publication of a paper on the logic of content in comparison with the systems **R** and **E** of relevant logic,
- Publication of a paper concerning the interpretation of justification logic as a logic of verification and falsification and the development of the notion of content within this logic,
- Publication of a paper regarding the applications of the logic of content. The paper may be either on Hempel's confirmation theory or on the logic of truth-making depending on which application will prove to be more fruitfull,
- Organization of a weekly postgraduate seminar on non-classical logics at the Department of Philosophy at USP,

- Organization of a series of workshops UNICAMP-USP on relevance, paraconsistency, and provability.

7 Candidate's profile

The candidate as a well-founded expectation to be able to develop the described project within two years in virtue of the following skills/records:

- A PhD dissertation focused on both the notion of propositional content and possible operations on this notion,
- A solid background in non-classical logics developed during four years spent at the Centro de Lógica, Epistemologia e História da Ciência at Universidade Estadual de Campinas,
- Publication of an article on an international journal (qualis A1) related to the conceptual background required by the project (Ciccarelli, 2019)

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